Limits — problems only

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Summary: This document contains some of the most common limits problems for you to review! Feel free to jump around or start from the beginning! Visit *https://sciency.tech* for the solutions and other problem-and-solution guides!

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1 How to read limits out loud

1. How do you read $f(x)$?
2. How do you read $\lim_{x \to a} f(x) = L$?
3. How do you read $\lim_{x \to a^-} f(x)$?
4. How do you read $\lim_{x \to a^+} f(x)$?

2 Basic limit problems

1. $\lim\limits_{x \to 3} \; x = ?$
2. $\lim\limits_{x \to a} \; (x^2 + 7) = ?$
3. $\lim\limits_{x \to \pi} \; \cos\left(\dfrac{x}{2}\right) = ?$
4. $\lim\limits_{x \to \infty} \; e^{-x} = ?$
5. $\lim\limits_{x \to a} \; \dfrac{x - 3}{x^2 + 7} = ?$
6. $\lim\limits_{x \to \pi} \; x \cos x = ?$

3 One-sided limits

1. Let
$$f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ 3x-7, & \text{if } x \geq 0 \end{cases},$$
then
$$\lim_{x \to 0^+} \; f(x) = ?$$

2. Let
$$f(x) = \begin{cases} x+2, \text{ if } x < 0 \\ 3x-7, \text{ if } x \geq 0 \end{cases}$$
then
$$\lim_{x \to 0^-} \; f(x) = ?$$

3. Let
$$f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ 3x-7, & \text{if } x \geq 0 \end{cases},$$
then
$$\lim_{x \to 0} \; f(x) = ?$$

4 Limit laws

1. $\lim\limits_{x \to a} \ (f(x) + g(x)) = ?$
2. $\lim\limits_{x \to a} \ f(x) \ g(x) = ?$
3. $\lim\limits_{x \to a} \ \dfrac{f(x)}{g(x)} = ?$
4. $\lim\limits_{x \to a} \ f\left(g(x)\right) = ?$
5. $\lim\limits_{x \to a} \ 17 = ?$
6. $\lim\limits_{x \to a} \ \left(f(x)\right)^2 = ?$
7. $\lim\limits_{x \to a} \ \left(f(x)\right)^n = ?$
8. $\lim\limits_{x \to a} \ (7x - 2)^3 = ?$
9. $\lim\limits_{x \to 0} \ \sqrt{x + 4} = ?$
10. $\lim\limits_{x \to \ -7} \ \sqrt{x + 4} = ?$

5 Harder limit problems

1. $\lim_{x\to 5} \dfrac{x^2-25}{x-5} = ?$

2. $\lim_{x\to \infty} \dfrac{1}{x} = ?$

3. $\lim_{x\to \infty} \dfrac{1}{x^2} = ?$

4. Let $a_n = 2 + 1/n$. Then $\lim_{n\to\infty} a_n = ?$

5. $\lim_{x\to 0} \dfrac{\sqrt{x^2+49}-7}{x^2} = ?$

6. $\lim_{x\to 5} \dfrac{\sqrt{x^2+24}-7}{x^2-25} = ?$

7. $\lim_{x\to \infty} \dfrac{\sqrt{4x^4+24x-7}}{x^2-25} = ?$
   (Try calculating this limit **without** using l'Hôpital's rule.)

8. $\lim_{x\to -\infty} \dfrac{5x^3+4x+7}{25-2x^3} = ?$

   (Try calculating this limit **without** using l'Hôpital's rule.)

6 l'Hôpital's rule

1. What is l'Hôpital's rule?
2. $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = ?$
3. $\lim\limits_{x\to -3} \dfrac{(x+3)^3}{x^2+9} = ?$
4. $\lim\limits_{x\to \infty} \dfrac{e^x}{x^2+4} = ?$