Limits — Limit laws

brought to you by *sciency.tech*

last updated: February 13, 2019

Summary: This document contains the problems demonstrating limit laws and their solutions.

Visit *https://sciency.tech* for other problem-and-solution guides!

Limit Laws

1. $\lim\limits_{x\to a}\ (f(x)+g(x)) = ?$

Solution: $\lim\limits_{x\to a}\ (f(x)+g(x)) = \lim\limits_{x\to a}\ f(x) + \lim\limits_{x\to a}\ g(x).$

2. $\lim\limits_{x\to a}\ f(x)\ g(x) = ?$

Solution:

$$\lim_{x\to a}\ (f(x)\ g(x)) = \left(\lim_{x\to a}\ f(x)\right)\left(\lim_{x\to a}\ g(x)\right).$$

3. $\lim\limits_{x\to a}\ \dfrac{f(x)}{g(x)} = ?$

Solution:

$$\lim_{x\to a}\ \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a}\ f(x)}{\lim\limits_{x\to a}\ g(x)}.$$

4. $\lim\limits_{x \to a} \; f\left(g(x)\right) = \, ?$

Solution:

$$\lim_{x \to a} \; f\left(g(x)\right) = f\left(\lim_{x \to a} g(x)\right),$$

assuming f is a continuous function.

5. $\lim\limits_{x \to a} \; 17 = \, ?$

Solution:

$$\lim_{x \to a} \; 17 = 17.$$

6. $\lim\limits_{x \to a} \; \left(f(x)\right)^2 = \, ?$

Solution:

$$\lim_{x \to a} \; \left(f(x)\right)^2 = \left(\lim_{x \to a} f(x)\right)^2.$$

7. $\lim\limits_{x \to a} \; \left(f(x)\right)^n = \, ?$

Solution:

$$\lim_{x \to a} \; \left(f(x)\right)^n = \left(\lim_{x \to a} f(x)\right)^n.$$

8. $\lim\limits_{x \to a} \; (7x - 2)^3 = \, ?$

Solution:

$$\begin{aligned} \lim_{x \to a} \; (7x - 2)^3 &= \left(\lim_{x \to a} \; (7x - 2)\right)^3 \\ &= (7a - 2)^3. \end{aligned}$$

9. $\lim_{x\to 0} \sqrt{x+4} = ?$

Solution: Since the square root function is continuous,

$$\begin{aligned} \lim_{x\to 0} \sqrt{x+4} &= \sqrt{\lim_{x\to 0} (x+4)} \\ &= \sqrt{4} \\ &= 2. \end{aligned}$$

10. $\lim_{x\to -7} \sqrt{x+4} = ?$

Solution:

$$\begin{aligned} \lim_{x\to -7} \sqrt{x+4} &= \sqrt{-7+4} \\ &= \sqrt{-3} \\ &= i\sqrt{3}, \end{aligned}$$

where $i = \sqrt{-1}$ is called the "imaginary number."