Limits — l'Hôpital's rule

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Summary: This document provides you a few problems demonstrating l'Hôpital's rule and their solutions

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l'Hôpital's rule

1. What is l'Hôpital's rule?

Solution: L'Hôpital's rule is a method that lets us use derivatives in evaluating limits involving "indeterminate forms," i.e. when a straight-forward approach gives us $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$.

More specificially, l'Hôpital's rule tells us that when

$$\frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty},$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

where the primes (') signify taking the derivative with respect to x.

2. $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = ?$

Solution: Since $\dfrac{\sin 0}{0} = \dfrac{0}{0}$, we apply l'Hôpital's rule:

$$\begin{aligned}\lim_{x\to 0} \frac{\sin x}{x} &= \lim_{x\to 0} \frac{(\sin x)'}{(x)'} \\ &= \lim_{x\to 0} \frac{\cos x}{1} \\ &= \cos 0 \\ &= 1.\end{aligned}$$

3. $\lim\limits_{x\to -3} \dfrac{(x+3)^3}{x^2+9} = ?$

Solution: Since

$$\frac{\lim\limits_{x\to -3} (x+3)^3}{\lim\limits_{x\to -3} (x^2+9)} = \frac{0}{0},$$

we apply l'Hôpital's rule:

$$\begin{aligned}\lim_{x\to -3} \frac{(x+3)^3}{x^2+9} &= \lim_{x\to -3} \frac{3(x+3)^2}{2x} \\ &= \frac{0}{-6} \\ &= 0.\end{aligned}$$

4. $\lim\limits_{x\to \infty} \dfrac{e^x}{x^2+4} = ?$

Solution: Since

$$\frac{\lim\limits_{x\to \infty} e^x}{\lim\limits_{x\to \infty} (x^2+4)} = \frac{\infty}{\infty},$$

we apply l'Hôpital's rule, which gives us

$$\begin{aligned}\lim_{x\to \infty} \frac{e^x}{x^2+4} &= \lim_{x\to \infty} \frac{(e^x)'}{(x^2+4)'} \\ &= \lim_{x\to \infty} \frac{e^x}{2x}.\end{aligned}$$

Since

$$\frac{\lim\limits_{x\to\infty} e^x}{\lim\limits_{x\to\infty} 2x} = \frac{\infty}{\infty},$$

we apply l'Hôpital's rule a second time:

$$\begin{aligned}
\lim_{x\to\infty} \frac{e^x}{2x} &= \lim_{x\to\infty} \frac{(e^x)'}{(2x)'} \\
&= \lim_{x\to\infty} \frac{e^x}{2} \\
&= \frac{\infty}{2} \\
&= \infty.
\end{aligned}$$

And so, the answer is

$$\lim_{x\to\infty} \frac{e^x}{x^2+4} = \infty.$$